



# Evaluation of an artificial market approach for GHG emissions trading analysis

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## ABSTRACT

In this study, an artificial market of greenhouse gases emissions trading is constructed applying a multi-agent model, and the validity of the approach is evaluated by comparing with a conventional method (a regression model) using real emissions trading market data. Mean errors, absolute mean errors, and root mean square errors are used for the examination. As a result of the comparison, it is shown that the proposed model has more power of explanation and is more effective in predicting the emissions trading price than the conventional approach.

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## 1. Introduction

Recently, emissions rights, also known as emissions allowances or credits, of greenhouse gases (GHG) receive attention as a financial commodity like stocks due to necessity to address GHG emissions abatement, and GHG emissions trading is gradually activated throughout the world. Although a variety of studies on emissions trading have been done, almost all of them focus on the final effects on economy and emissions abatement from theoretical perspectives using economic models such as applied general equilibrium models [29]. On the other hand, there are very few studies focusing on the trade process. Because emissions rights are traded on a daily basis in the market like stocks and foreign exchange, the trade process is substantially important as well as the final trade outcomes. Moreover, understanding of the process would highly influence on the effects of emissions trading.

During this decade, an artificial market approach has gradually started to be used for economic analysis, instead of conventional approaches to address complicated dynamic systems. Since then, it has been frequently applied to analysis of financial markets such as foreign exchange markets and stock markets, from conceptual studies to application studies (e.g. [1,2,4,8–11,14–17,19,24,25,27,30,31] and many other studies). The artificial market in this study is a virtual market constructed on the computer as a multi-agent model. Such model is based on heterogeneous, adoptive agents like the studies above, although the degree of the heterogeneity in the proposed model is not so large. In such models, multiple bounded rational agents construct a financial market with a bottom-up approach. For example, Arthur et al. [2], LeBaron [16], and LeBaron et al. [18] constructed an agent-based (a multi-agent) artificial financial model named The Santa Fe Artificial Stock Market and analyzed phenomena in and properties of stock markets. In their studies, a stock market was constructed and the dynamic features were emerged through interactions of artificial adoptive agents who learn and modify their beliefs using a

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genetic algorithm (GA). By applying an artificial market approach where trade is based on micro agents behaving as traders in the computational market, it is possible to simulate various phenomena occurred in actual financial markets which are difficult or impossible to explain by conventional (mainstream) economic theories (such as the Walrasian equilibrium) [13,28]. It can also be used to analyze traders' behavior which is difficult to understand only from the actual trade. Because it is a multi-agent model in a competitive situation, each agent behaves to accomplish its purpose independently through interactions with other agents rather than to accomplish a single, shared purpose of all the agents. One of the most significant characteristics of this approach is that behavioral aspects of economic agents which have been considered important for their decision making in the actual markets can be incorporated into the model. On the contrary, the theory of rational expectations [26], which is the mainstream hypothesis used in economic theories so far, imposes strong assumptions that homogeneous agents act under the perfect foresight and do not need to learn or modify their behavior at all. Thus, such features are not taken into account there. However, in real situations, heterogeneous agents exist, and they are bounded rational, have imperfect information, and change the ways of decision making continuously by adapting to the environment. Thus, it is required to consider the behavioral views.

Although the artificial market approach has such a big advantage, there are some criticisms that the theoretical rationale is poor [13,28], hence the approach has not been so popular in economics yet. However, it is also true that some studies indicate the effectiveness as mentioned above. The purpose of this study is to evaluate effectiveness of an artificial market approach (or a multi-agent model for market simulations) for GHG emissions trading analysis. This study is based on the concept provided by Izumi and Okatsu [9] and Izumi and Ueda [10,11]. They analyzed dynamics of a foreign exchange market applying an artificial market approach to examine the effectiveness of the approach in analyzing economic phenomena. They proposed a model with artificial adoptive agents learning and modifying their belief systems using GA. In their model, not only trend data but also fundamentals data were used for the agents' decision making. After comparing with existing approaches (a linear regression model and a random walk model) by mean absolute errors and root mean square errors, they concluded that their model could forecast and explain the actual exchange rate more accurately.

The structure of the following sections is as follows: GHG emissions trading is explained briefly in the second section. In the third section, the general framework of the proposed model is explained. In the fourth section, the structure of the model is described. In the fifth section, the framework of the analysis is explained. In the sixth section, the evaluation methods and the results of this study are shown. Finally, the seventh section includes discussions and concluding remarks.

## 2. GHG emissions trading

GHG emissions trading is one of the policy approaches to efficiently abate GHG emissions by providing economic incentives and it is expected to play an important role as a climate change policy. This method has attracted attention since it was stipulated in the Kyoto Protocol (Article 17)<sup>1</sup>. As a result, it is operated and planned to operate in some countries and regions. For example, there are EU Emissions Trading Scheme, UK Emissions Trading Scheme, emissions trading in Denmark, Chicago Climate Change in US, Regional Greenhouse Gas Initiative in US, Western Regional Climate Action Initiative in US, Japanese Voluntary Emissions Trading Scheme, and NSW Greenhouse Gas Reduction Scheme in Australia. Furthermore, many kinds of emissions trading systems are proposed in the US Congress. As there are two types of systems, "cap and trade" and "baseline and credit", the former system is more popular and also generates more emissions rights. It is possible to use both systems at the same time like emissions trading and CDM/JI under the Kyoto Protocol. Although carbon tax is another method to efficiently abate GHG emissions, emissions trading is superior in order to achieve a certain abatement amount.

A simple process of a cap and trade system can be explained as follows: a regulator determines the scope of the system (regions, sectors, and so on) and sets a limit or cap on the total emissions. Emissions rights are allocated to economic agents in the scope based on a certain standard and each economic agent is required to hold the rights equal to its emissions. If an economic agent does not hold enough emissions rights, it has to abate its emissions by itself<sup>2</sup> and/or buy emissions rights from agents who emit less than the emissions rights they have in order to comply with the rule. Normally, it selects the cheapest way. Therefore, in theory, since economic agents who can abate emissions most cheaply do so, emissions abatement is achieved at the lowest cost. The trade price is determined to balance supply of and demand for emissions rights. The supply and demand are influenced by various factors such as energy prices, economic conditions, and climate. For example, if summer temperatures are higher than usual, demand for emissions rights will grow as a result of increase in energy consumption for air conditioning, and the price will rise accordingly.

Agents who participate in emissions trading usually make decisions on trade and emissions abatement as mentioned above. However, in the actual emissions trading, agents who participate only for trade without abatement exist, thus there are more agents who participate in trade than those who implement emissions abatement. Therefore, although the primary purpose of emissions trading is of course to realize efficient emissions abatement, trade irrelevant to the efficiency such as speculative behavior exists<sup>3</sup>. As it will be mentioned in the following section, this study focuses on the trade part.

<sup>1</sup> See [http://unfccc.int/kyoto\\_protocol/mechanisms/emissions\\_trading/items/2731.php](http://unfccc.int/kyoto_protocol/mechanisms/emissions_trading/items/2731.php) for more details on emissions trading under the Kyoto Protocol.

<sup>2</sup> Emissions can be abated by introducing energy-efficient technologies and reducing production. Also, increase of carbon sinks can be used to offset GHG emissions. Carbon capture and storage will be a practical approach to offset the emissions in the near future.

<sup>3</sup> Due to such behavior, it is contemplated that the trade price is raised.

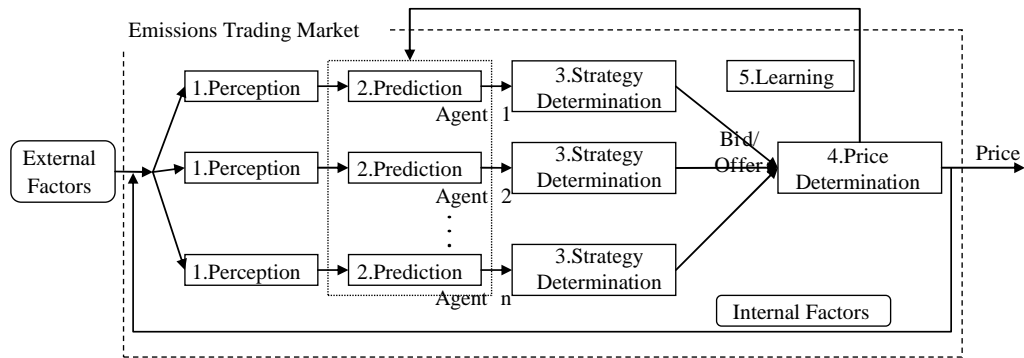


Fig. 1. General framework.

### 3. General framework

As observed in actual emissions trading markets, emissions rights are traded similar to foreign exchange in exchange markets and stocks in stock markets. A lot of existing (conventional) economic models of such markets usually include the following four steps explicitly or implicitly, thus these steps can also be applied to emissions trading markets.

First, each agent (trader) predicts trade price for the current period from the related data. Also, it determines expected trade amount which it hopes to trade in the period. Then, it either bids or offers to the market. Each agent's decision making process consists of the following three steps.

- (1) Perception step: perception of raw data (factors).
- (2) Prediction step: prediction of change in trade price for the current period based on its belief (a prediction method or weights on the factors) and the perceived factors.
- (3) Strategy determination step: determination of a strategy to buy or sell emissions rights and expected trade price and amount, and submission of a bid or offer message to the market.

As a result of this decision making process, bids and offers are gathered in the market. By aggregating the bids and offers respectively, a demand curve and a supply curve are formed.

- (4) Price determination step: determination of equilibrium trade price based on the two curves.

In addition to the four steps above, the proposed model includes another step to prepare for the next trade, which is not considered in conventional models, after trade price is determined and trade is implemented.

- (5) Learning step: modification of agents' belief systems to improve the prediction accuracy for the next and further trade.

The general framework is also shown in Fig. 1.

The more detailed descriptions will be given in the next section.

### 4. Structure of the model

The proposed model is composed of a number of agents who trade emissions rights in an emissions trading market. Each agent holds some money and emissions rights, and intends to hold an optimum amount of assets. One period in the model corresponds to one week in the real world. The following five steps are implemented in one period.

The model is constructed using *artisoc*<sup>4</sup>.

#### 4.1. Perception step

First, each agent collects raw data and perceives factors which are made by interpreting the raw data to predict the price change. It is assumed that all agents have the same raw data and factors in this study. The number of factors is 11 (Table 1) and they can be classified into external factors (No. 1–8), which are external information of the market and affect the trade price from outside the market, and internal factors (No. 9–11), which are trends of the trade price that come from the market. The former is defined as the factors related to economic and relative data in the real world. The latter is defined as the factors related to the price change in the market. The external factors are generated by coding the raw data between  $-3$  and  $+3$  (continuous values) according to the degree of the data. For the internal factors, the raw data are used directly. In this study,

<sup>4</sup> *Artisoc* is a multi-agent simulator developed by Kozo Keikaku Engineering Inc. See <http://mas.kke.co.jp/modules/tinyd0/index.php?id=1> for the detail. Although it is a Japanese page, there are some English manuals.

**Table 1**  
Prediction factors

	Factors	Raw data
1	Energy price 1	Petroleum price
2	Energy price 2	Coal Price
3	Energy price 3	Natural gas price
4	Temperature 1	Heating degree days
5	Temperature 2	Cooling degree days
6	Energy consumption 1	Fossil fuels
7	Energy consumption 2	Renewable energy
8	Production	Industrial production
9	Shorter trend 1	Price change in the last week ( $\Delta p_{t-1}$ )
10	Shorter trend 2	Change of 9 ( $\Delta p_{t-1} - \Delta p_{t-2}$ )
11	Longer trend	Price change in five weeks ( $p_{t-1} - p_{t-6}$ )

\* All data are statistical data in US.

\*\* All external factors (No. 1–8) are the factors regarded as market drivers in Chicago climate exchange (CCX). See <http://www.chicagoclimateexchange.com/> and Section 5.2 below.

\*\*\*  $t$  is time (period),  $p_t$  is trade price in time  $t$  (in logarithms),  $\Delta p_t$  is price change in time  $t$  (in logarithms).

not only internal factors (data) but also external factors which have not been used or considered in the previous studies on emissions trading applying multi-agent models [20–23] are used. Such external factors also function as important drivers of agents' decision making, hence it can be said that the agents in this model consider more about the economic structure than those in the previous studies<sup>5</sup>.

#### 4.2. Prediction step

Next, each agent predicts the price change using the factors above. In this study, each agent has its own weight on each factor for the prediction<sup>6</sup>. The weights are set into 13 levels ( $\pm 3, \pm 2, \pm 1, \pm 0.8, \pm 0.5, \pm 0.1$ , and 0). The price change is predicted by the following formula<sup>7</sup>.

$$E_{it}[\Delta p_t] = \alpha \left( \sum_{j=1}^{11} w_{ijt} x_{jt} \right) \tag{1}$$

where  $i$  is an agent,  $j$  is a factor,  $E_{it}[\Delta p_t]$  is agent  $i$ 's prediction on  $\Delta p_t$ ,  $w_{ijt}$  is a weight agent  $i$  assigns to factor  $j$  in time  $t$ ,  $x_{jt}$  is a value for factor  $j$  in time  $t$ ,  $\alpha$  is a scale parameter.

Eq. (1) suggests that in the case of  $w_{ijt} x_{jt} > 0$ , it causes the rise in the price and in the case of  $w_{ijt} x_{jt} < 0$ , it causes the drop in the price.

Also, variance of the prediction is calculated from the following formula [9–11].

$$V_{it}[\Delta p_t] = \frac{1}{\sqrt{|(wx_+)^2 - (wx_-)^2|}} \tag{2}$$

where  $V_{it}[\Delta p_t]$  is variance of agent  $i$ 's prediction on  $\Delta p_t$ ,  $wx_+$  is summation of  $w_{ijt} x_{jt} > 0$ ,  $wx_-$  is summation of  $w_{ijt} x_{jt} < 0$ .

The variance is inversely proportional to the coherence of the prediction. Therefore, the smaller the variance is, the higher the confidence in the prediction will be.

It is obvious that both  $E_{it}[\Delta p_t]$  and  $V_{it}[\Delta p_t]$  are different among agents and among periods.

#### 4.3. Strategy determination step

After the prediction step, each agent determines its bid or offer price and amount based on the prediction in order to maximize the utility. The optimum amount of emissions rights to possess is calculated from the following formula (see Izumi and Ueda [11] for the detail).

<sup>5</sup> Based on the surveys and interviews with traders, Izumi and Okatsu [9] and Izumi and Ueda [11,12] suggest that it is possible to consider the economic structure and each agent's mind model of the market structure by incorporating fundamentals factors (external factors in this paper) into the model. In addition, Muchnik et al. [25] points out that it is necessary to consider external events such as economic fundamentals to improve market models further. From these studies, it is inferred that external factors are also the important drivers for agents participating in emissions trading since it is similar to other financial trading.

<sup>6</sup> The weights represent agents' heterogeneity. Although the degree of the heterogeneity in the proposed model is not large as mentioned above, it is possible to model various kinds of traders such as fundamental traders and chartists. For example, if the weights on the internal factors are zero and at least one of the weights on the external factors is nonzero, the agent becomes a fundamental trader. In addition, even among fundamental traders, the characteristics (response to the factors) are quite different according to the degree of the weights.

<sup>7</sup> In order to consider the price change as the rate against the price in the previous period, the change and price are represented by logarithms. It is based on the thought that 1¢ change from \$1/t-CO<sub>2</sub> and 2¢ change from \$2/t-CO<sub>2</sub> are equivalent.

<sup>8</sup> It is different from "variance" as a statistical term.

$$q_{it}^* = \beta \frac{E_{it}[\Delta p_t]}{V_{it}[\Delta p_t]} \quad (3)$$

where  $q_{it}^*$  is the optimum amount agent  $i$  possesses in time  $t$ ,  $\beta$  is a scale parameter.

In order to gain the optimum amount, each agent bids or offers the amount equivalent to the difference between the optimum amount  $q_{it}^*$  and the holding amount  $q_{it}$ . That is, each agent's bid or offer amount is  $\Delta q_{it} = q_{it}^* - q_{it}$ . In the case of  $\Delta q_{it} > 0$ , it bids to buy emissions rights. On the other hand, in the case of  $\Delta q_{it} < 0$ , it offers to sell. Each agent's strategy is to approach the amount possessed to the optimum by buying (selling) emissions rights when the optimum amount is larger (smaller) than the amount possessed and the actual trade price is lower (higher) than expected.

Concerning the bid or offer price, each agent uses the predicted price change. That is, the bid or offer price (in logarithms) is defined as  $ep_{it} = p_{t-1} + E_{it}[\Delta p_t]$ .

Then, each agent sends the bid or offer message to the market.

#### 4.4. Price determination step

When the messages from all agents are sent, a demand curve and a supply curve are formed by aggregating bids and offers respectively. Then, the intersecting point of the two curves is considered the trade price for this period. As a result, trade is implemented among agents who expected to buy emissions rights above the equilibrium price and sell below the price.

#### 4.5. Learning step

In this model, the rational expectations hypothesis is not considered and the bounded rationality is considered instead. That is, each agent has its own prediction method represented as the weights on the factors mentioned in Section 4.2. It means that the difference of the prediction methods is represented by the difference of the weights. Each agent uses GA [6,7] to adapt the weights in this study. GA is often adopted as a learning method in multi-agent models for market simulations and economic situations (for example, see Brenner [3], Duffy [5], and LeBaron [17]). When GA is used in the proposed model, a string  $w_{it}$  consists of all the weights of one agent  $w_{ijt}$ , namely  $w_{it} = (w_{i1t}, w_{i2t}, \dots, w_{i11t})$ . By using GA operations, selection, crossover, and mutation, each agent learns and changes the string. Izumi and Okatsu [9] and Izumi and Ueda [11] found from their surveys and interviews with traders that there were similarities between the way agents learn in the financial market and GA (or adaptation in the ecosystem). They suggest that agents always replace the old belief systems with new ones to improve the prediction, learn from their failures, imitate other agents' belief systems with some degree of reproductive accuracy, and change the belief systems by communicating with other traders or by itself in the market.

First, each evaluation value on the string is calculated from the following formula.

$$EV_{it}[w_{it}] = \max_i (|E_{it}[\Delta p_t] - \Delta p_t|) - |E_{it}[\Delta p_t] - \Delta p_t| + 1 \quad (4)$$

where  $EV_{it}[w_{it}]$  is an evaluation value on agent  $i$ 's string of weights in time  $t$ . Adding 1 in Eq. (4) is to leave a low possibility to select the least evaluated string.

As Eq. (4) shows, the evaluation is based on the expected and actual price. The more precisely the price is predicted, the higher the evaluation will be.

GA operations are as follows.

**Selection:** It is an operation that an agent  $i$  selected with a probability  $sp$  (i.e. "total agents  $\times sp$ " agents are selected) copies a string of weights of another agent  $k$  who is selected according to the probability proportionally to the evaluation  $EV_{it}[w_{it}]$ . That is, each agent does not change its weights if the prediction is precise and replaces its weights with others if the prediction is relatively less precise. This is a process corresponding to the change in a prediction method (a belief) by imitating a successful trader in actual markets [9,11]. Consequently, belief systems with low evaluation disappear.

**Crossover:** It is an operation that an agent  $i$  selected with a probability  $cp$  (i.e. "total agents  $\times cp$ " agents are selected) follows a part of the string of another agent  $k$  who is selected randomly. One-point crossover is adopted and the crossover point is determined randomly. This is a process corresponding to the change in a prediction method by exchanging information through communication with another trader [9,11].

**Mutation:** It is an operation that an agent  $i$  selected with a probability  $mp$  (i.e. "total agents  $\times mp$ " agents are selected) changes one of its weights randomly. This is a process corresponding to setting a new value by each trader's individual thought [9,11]. Therefore, it is implemented individually without interactions unlike the other operations.

## 5. Analysis

### 5.1. General framework

Simulation analysis in this study is composed of training periods, which are for training of agents, and experiment periods, which are for analysis (Fig. 2). The training periods and experiment periods are continual. During the training periods

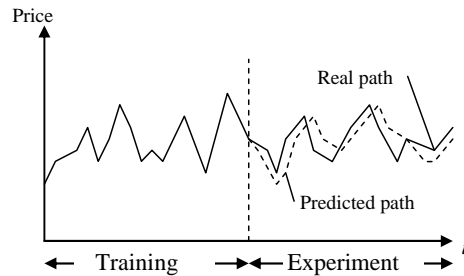


Fig. 2. Model simulation.

before the experiment periods start, each agent trains the weights using the actual trade price and prediction factors (see Table 1) for the corresponding periods. While training is implemented, the price determination step (see Section 4.4) is skipped and a series of the actual trade price is used instead. The training periods are repeated several times using the same data set and the weights of each agent initially generated randomly are improved through the training. After the training, an emissions trading simulation is implemented in the experiment periods using the prediction factors (see Table 1) for the corresponding periods and the weights trained in the training periods. During the experiment, trade price is determined through the price determination step.

In both periods, period run consists of “Perception step (Section 4.1) → Prediction step (Section 4.2) → Strategy determination step (Section 4.3) → Price determination step (Section 4.4) → Learning step (Section 4.5)” except for the price determination step in the training periods.

## 5.2. Setup of analysis

There are only a few emissions trading markets operated officially and the history is very short. Therefore, it is difficult to collect the long-term data. In this study, Chicago Climate Exchange (CCX) which started in 2003 is taken up and the 2003 vintage data are used. CCX is the world’s first and North America’s only legally binding rules-based GHG emissions trading system although entry to the system is voluntary. It is also the only system for emissions trading based on the six GHG in the Kyoto Protocol. The system is mainly cap and trade, but it also includes baseline and credit. There are a wide range of members including industrial and public sectors, and a lot of participants just for trade exist as well in the market. Therefore, in this analysis, we focus on the trade part which is the common aspect for all the participants<sup>9</sup>.

First, one year between December 2003 and December 2004 (52 weeks) is used as the training periods. Since one period in the simulation corresponds to one week, the training continues 52 periods. The training is repeated 20 times for each. Then, one year between December 2004 and December 2005 (also 52 weeks) is used as the experiment periods. The total number of simulations is 200. There are 100 agents in the market.

## 6. Evaluation of the model

### 6.1. Methods

In order to evaluate the proposed artificial market model, it is compared with a linear multiple regression model<sup>10</sup>. The regression model, a popular prediction method in economics and social science, is a statistical technique to model the relationship between a dependent variable and explanatory variables which are the elements considered to influence on the dependent variable. Thus, it is used to predict the dependent variable from the explanatory variables and estimated parameters. In this study, trade price is predicted using multiple factors which are considered to influence on the price, thus a linear multiple regression model is applied for the comparison.

The two models are compared quantitatively using actual data as described above. To construct the regression model, the same factors used in the artificial market model (see Table 1) are treated as the explanatory variables and the trade price is treated as the dependent variable. Also, the same training and experiment periods are applied. Using the explanatory vari-

<sup>9</sup> It is also considerably hard to consider the abatement cost part because of difficulty to estimate marginal emissions abatement cost functions especially for companies and organizations.

<sup>10</sup> Although analyses using a random walk model (a particular type of time-series models) and some time-series models were tried, results with high accuracy were not obtained. Therefore, these results are not explained here. The failure of the former analysis suggests that the trade price is not determined randomly but determined by some sort of factors (as in the analysis in this study). The failure of the latter analyses suggests that strong temporal structure is not seen in the data, that is, intertemporal regularity in the data is weak.

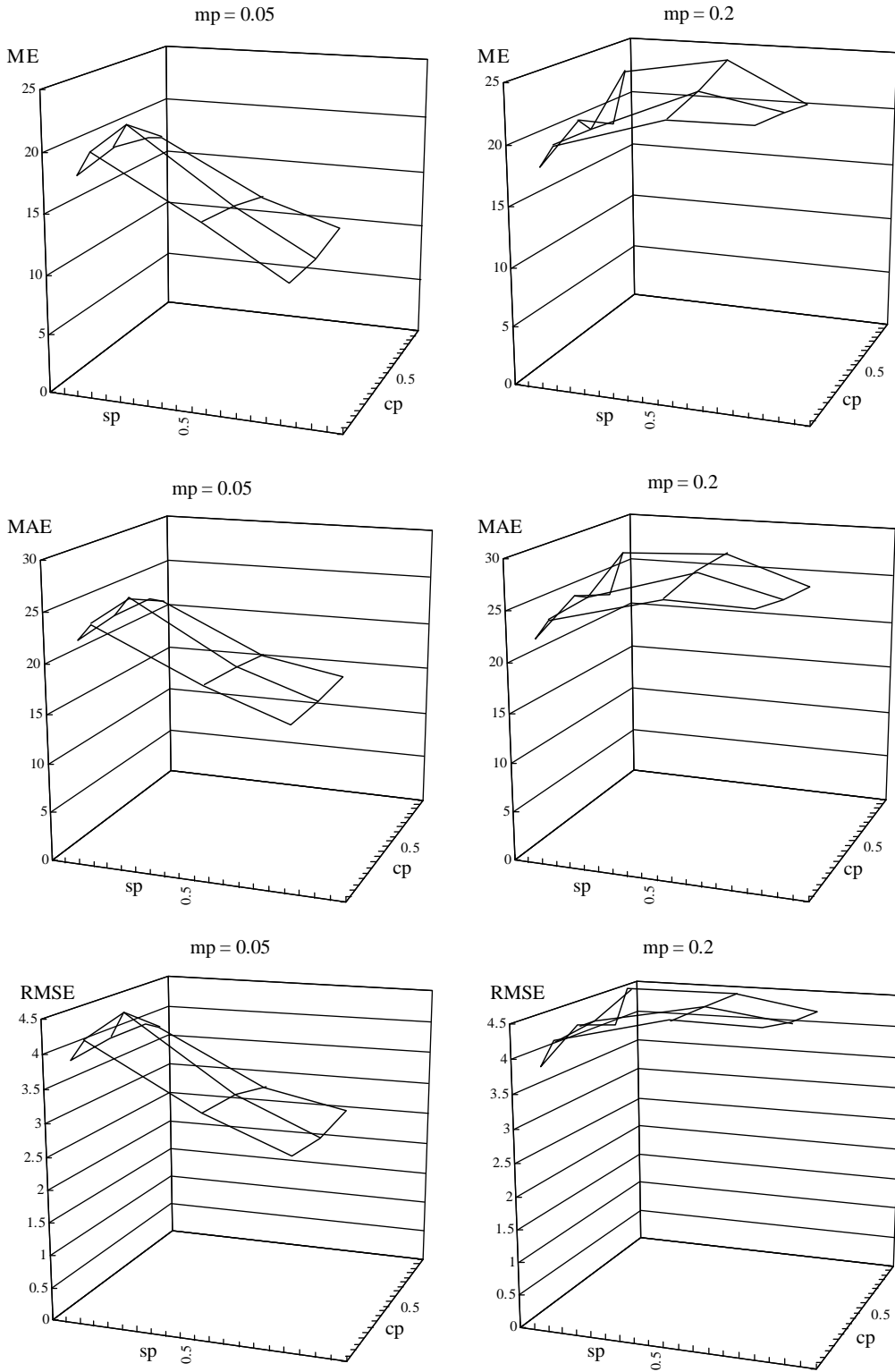


Fig. 3. ME, MAE, and RMSE under different parameter sets ( $m = 52$ ).

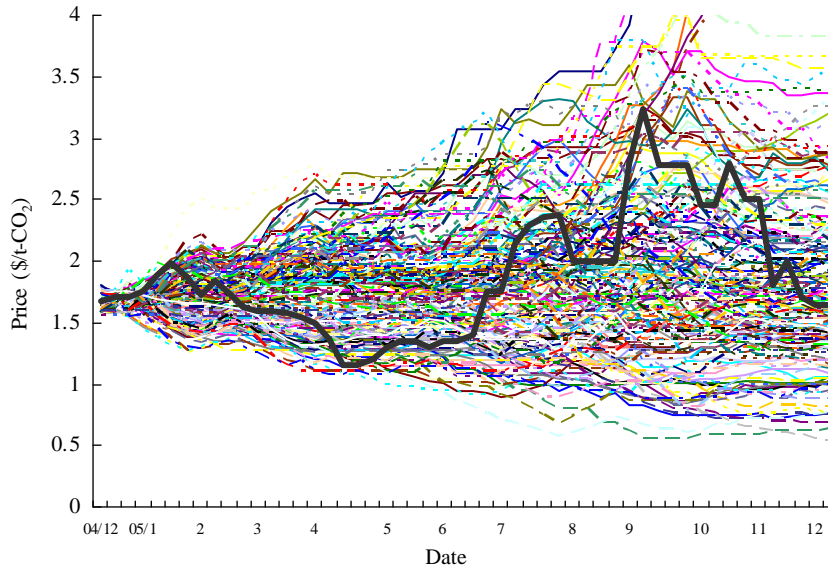
ables and estimated parameters, the trade price for  $m = 1, 2, 4, 13, 26,$  and  $52$  weeks ahead from the end of the training periods are estimated. In the same way, the artificial market model predicts the trade price in the same periods above.



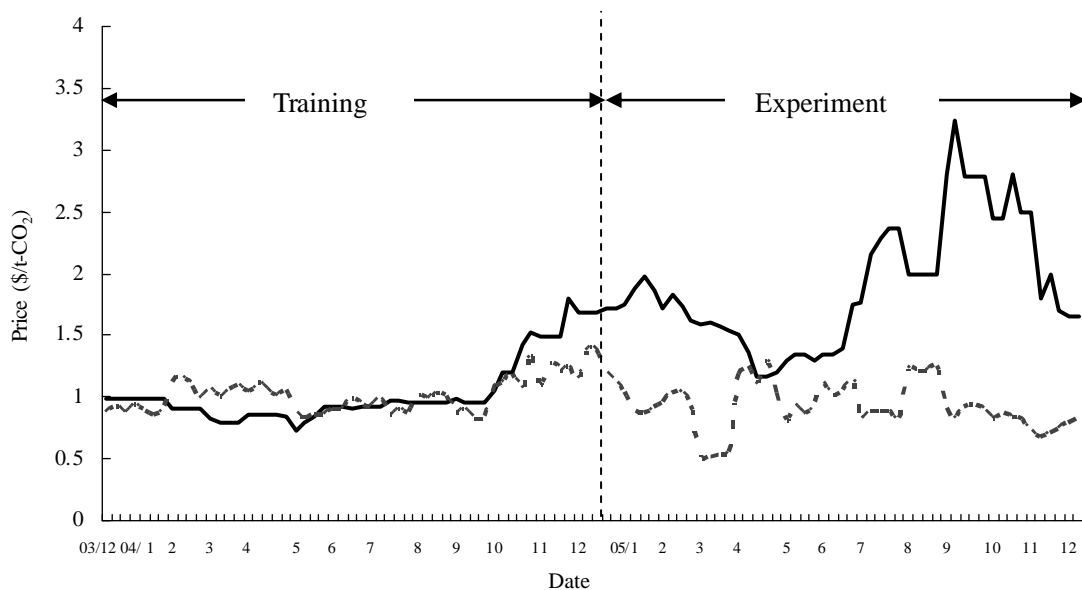
**Table 2**  
ME, MAE, and RMSE of the artificial market model and the regression model

m	ME		MAE		RMSE	
	Artificial market	Regression	Artificial market	Regression	Artificial market	Regression
1	0.027 (-85.9)	0.194 (0)	0.027 (-85.9)	0.194 (0)	0.027 (-85.9)	0.194 (0)
2	0.048 (-91.1)	0.545 (0)	0.051 (-90.6)	0.545 (0)	0.040 (-90.1)	0.401 (0)
4	0.099 (-93.3)	1.488 (0)	0.131 (-91.2)	1.488 (0)	0.075 (-90.4)	0.781 (0)
13	0.839 (-89.9)	8.272 (0)	1.128 (-86.4)	8.272 (0)	0.387 (-84.4)	2.483 (0)
26	2.874 (-77.7)	12.871 (0)	4.180 (-68.1)	13.098 (0)	1.029 (-66.2)	3.044 (0)
52	9.827 (-72.6)	35.872 (0)	14.57 (-59.6)	36.099 (0)	2.614 (-53.6)	5.632 (0)

\* "Artificial market" shows the values for the minimum case.  
\*\* In parentheses are percentage differences relative to "Regression".



**Fig. 4.** Simulation paths of artificial market simulations. \* The bold line represents the actual price.



**Fig. 5.** A path of regression analysis. \* The solid line represents the actual price and the dashed line represents the regression analysis.



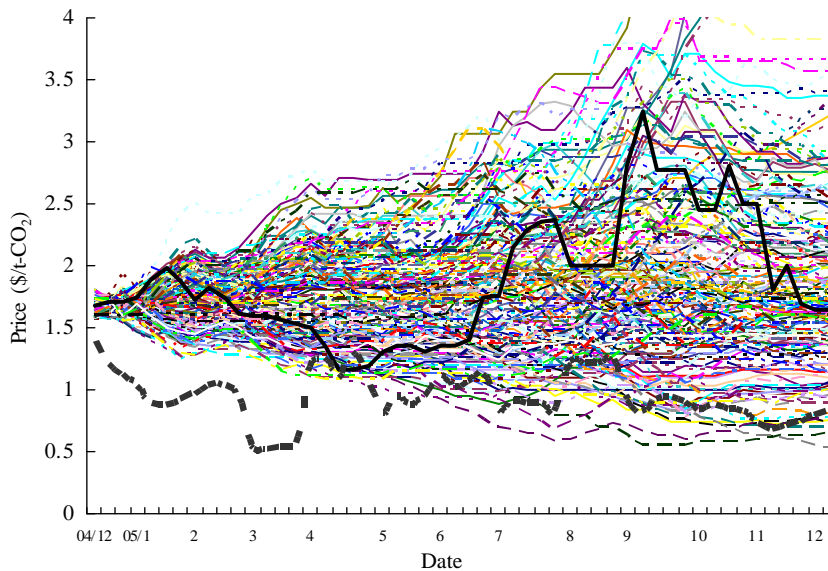


Fig. 6. Integration of Figs. 4 and 5. <sup>\*</sup> The solid bold line represents the actual price and the dashed bold line represents the regression analysis.

Comparison of the results is implemented by the following methods: mean error (ME), mean absolute error (MAE), and root mean square error (RMSE). They are calculated from the following formulae<sup>11</sup>.

$$ME = \frac{\sum_{s=1}^N \sum_{h=1}^m (p_h^* - p_h)}{N} \quad (5)$$

$$MAE = \frac{\sum_{s=1}^N \sum_{h=1}^m |p_h^* - p_h|}{N} \quad (6)$$

$$RMSE = \frac{\sum_{s=1}^N \left\{ \sum_{h=1}^m (p_h^* - p_h)^2 \right\}^{1/2}}{N} \quad (7)$$

where  $s$  is a simulation number,  $h$  is a prediction period from the end of training,  $p_h^*$  is actual trade price in prediction period  $h$  (in logarithms),  $p_h$  is predicted trade price in prediction period  $h$  (in logarithms),  $N$  is the total number of simulations.

## 6.2. Results

For the simulations,  $0.1 < sp < 0.8$ ,  $0.1 < cp < 0.8$ , and  $mp = 0.05$  and  $0.2$  are used as the GA parameters. The comparison results are shown in Fig. 3 and Table 2. They suggest that all of the indicators show more or less smaller errors in the case of the artificial market model independently of the prediction periods. As Table 2 shows, all ME, MAE, and RMSE of the market model is smaller than those of the regression model and they are smaller over 72.6%, 59.6%, and 53.6%, respectively in the smallest case. These results indicate that the artificial market model has more power of explanation and is more effective in predicting trade price in the emissions trading market than the regression model for the analyzed period and data. Further research has to show the adequacy of this approach.

Fig. 4 shows the simulation paths in the case of the smallest errors. In this case,  $sp$ ,  $cp$ , and  $mp$  are set as follows:  $sp = 0.8$ ,  $cp = 0.2$ , and  $mp = 0.05$ . Observing the individual results in detail, it is suggested that some predict well and others do not. Concerning the coefficient of correlation, 83 are significant of 1% level and other 14 are significant of 5% level out of 200 trials. On the other hand, in the case of the regression model, the coefficient of correlation during the experiment periods is 0.19 and it is not significant (Fig. 5). From this comparison, it is also indicated that the artificial market model predicts the trade price in the emissions trading market more precisely than the regression model (Fig. 6). This implies that emissions trading is influenced by the traders' behavior and interactions.

<sup>11</sup> In the case of the regression analysis, it is same as  $N = 1$  in Eqs. (5)–(7).

## 7. Concluding remarks

In this study, an artificial market of GHG emissions trading was constructed applying a multi-agent approach, and the power of explanation and prediction was compared with that of a linear multiple regression model. The result showed that the artificial market model could predict the trade price more accurately than the regression model for the analyzed period and data. Basically, the reason is considered as follows: regression models are just extrapolations based on past tendencies of a dependent variable and explanatory variables, thus it is difficult to predict phenomena without a certain relationship between the dependent variable and explanatory variables. On the other hand, artificial market models aim to attain the optimum state through agents' continual interactions and learning (adaptation), thus it is possible for the models to respond to changing situations more appropriately and predict phenomena without a steady pattern to some extent. From this study, it was also suggested that actual emissions trading was based on behavior of each individual trader but not based on the traditional economic assumptions.

Since artificial market models have been applied without evaluating the model validity in previous studies on emissions trading, it is considered that this study contributes to showing availability of the models to work in market analysis, and promotes more studies on this area using this methodology.

However, there are some shortage points in the proposed model due to the limitations. Therefore, there is room for improvement for the future studies. The points to improve can be considered from the aspect of agents and that of market. Considering the former, data agents use for prediction, factors related to agents' psychological aspects, and agents' learning method would be significant. It was assumed that all agents used the same data to predict trade price and amount in this study. However, some traders in the actual market must have their unique information and data for prediction, especially if they are professionals. Thus, investigation of what data they depend on and how they interpret the data would be valuable. Also, considering factors related to agents' psychological aspects, which can emphasize agents' heterogeneity by modeling each agent's individual response to prediction factors more precisely reflecting its internal thought, would be important to express diversity of agents further. Furthermore, although GA was used for the learning method in this study following the concept provided by the previous studies [9–11], there are other learning methods for economic learning [3]. Agents' learning is one of the critical elements to model artificial markets, thus more elaborate investigation on the methods would also be an important issue.

Considering the latter, it would be significant to improve the market condition to analyze under more realistic assumptions. Although this study focused on the trade part of emissions trading, emissions trading is affected by various factors such as other markets, laws and institutions, and many other socioeconomic systems. Thus, improvement such as incorporation of the emissions abatement part into the model and introduction of the institutions related to the emissions trading market would be necessary.

By upgrading the model in such a way, it is expected that the artificial market will be closer to the actual market and the prediction accuracy will be improved.

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